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The Future of Atomic Physics

P. A. M. Dirac

Florida State University, Tallahassee, Florida

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We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.

Many people are happy with this situation because it has a limited amount of success. But this is not good enough. *Physics must be based on strict mathematics.* One can conclude that the fundamental ideas of the existing theory are wrong. A new mathematical basis is needed.

The situation is comparable to that in the early 1920s, before Heisenberg made his breakthrough in 1925. We then had to work with the Bohr orbit theory. The work was successful for some problems, where there was just one electron in an atom that was mainly involved. But the success was very limited. One had no method of taking into account the interaction of the electrons in an atom. Strenuous efforts were made to remedy this defect by introducing a suitable interaction between the Bohr orbits. But these efforts led nowhere.

One can see now how hopeless these efforts were. To make real progress one needed a new mathematics, involving noncommutative algebra. The idea for this was provided by Heisenberg in 1925.

We are now again in the situation of needing some new mathematics. Many people who realize this are trying to extend and develop field theory. But it is doubtful whether such a development will go deep enough. Field theories, involving some kind of curved space, are needed if we are dealing with gravitation. But gravitation is excessively weak in atomic physics, so it seems unlikely that it will play an important role. Thus we should look elsewhere.

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We must concentrate on Einstein's special theory of relativity, not his general theory. Thus we should work with representations of the Lorentz group. These representations have been much studied in the past and you may think we know all about them. But this is far from true. All the irreducible representations are known, but in physics reducible representations can be important and the extra features that they bring in must be taken into account.

Let us consider first the irreducible representations. Each of them corresponds to some quantity with a definite transformation law under Lorentz transformations. Taking the successive representations, they correspond to scalars, vectors, and tensors of various ranks involving symmetries. Then there are the two-valued representations, involving quantities that change sign when one applies a rotation of one revolution about any spatial axis. These representations can all be combined into reducible ones.

But this does not exhaust all the representations. There are some further ones for which the matrices can be arranged like

$$\begin{array}{|c|c|}\hline A & 0 \\ \hline L & B \\ \hline\end{array} \tag{1}$$

where we have zero for all the elements in the top right rectangle, and the square parts A and B and the lower left rectangle L are left undetermined. If we multiply two such matrices together by matrix multiplication, we get another one, as is shown by

$$\begin{array}{|c|c|}\hline A_1 & 0 \\ \hline L_1 & B_1 \\ \hline\end{array}\;\begin{array}{|c|c|}\hline A_2 & 0 \\ \hline L_2 & B_2 \\ \hline\end{array} = \begin{array}{|c|c|}\hline A_3 & 0 \\ \hline L_3 & B_3 \\ \hline\end{array} \tag{2}$$

$$A_1 A_2 = A_3 \tag{3}$$

$$L_1 A_2 + B_1 L_2 = L_3 \tag{4}$$

$$B_1 B_2 = B_3 \tag{5}$$

Equation (3) shows that the A's form an ordinary representation of the Lorentz group. Similarly equation (5) shows that the B's also form an ordinary representation of the Lorentz group. Equation (4) is a new type of equation, and it shows that if we have a solution and we multiply the L's by any number λ (not zero) we get another solution.

To understand the significance of this kind of representation let us divide the wave function ψ into two parts ψ_A and ψ_B corresponding to the two parts in which the rows or columns of the matrices are divided, so ψ appears as

$$\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

When we multiply the matrix (1) into this wave function, we get

$$\text{new } \psi_A = A\psi_A$$

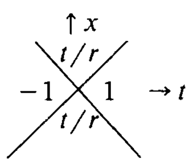
$$\text{new } \psi_B = L\psi_A + B\psi_B$$

Thus the new ψ_A is fixed by the original ψ_A, while both the original ψ_A and original ψ_B are needed to fix the new ψ_B. Thus a Lorentz transformation affects ψ_A by an amount depending only on ψ_A while it affects ψ_B by an amount depending on ψ_A and ψ_B. In short, ψ_A affects ψ_B while ψ_B does not affect ψ_A.

Now you may think that this is a completely unphysical situation. According to Newton, if A acts on B, then B will react on A. But Newton's laws apply to classical mechanics and things may be different in quantum mechanics. It could be that ψ_A represents a radioactive atom, capable of spontaneous disintegration, and ψ_B represents what is emitted by the disintegration. The original atom certainly influences the products of the disintegration, but these products move away and no longer have any influence on the emitting atom. This is just the physical situation which the mathematics provides.

Representations of the type (1) are called *pathological.* I would like to propose that such representations of the Lorentz group will be important in the physics of the future.

Let us set up a simple example of a pathological representation of the Lorentz group. We take solutions of the wave equation $\Box\psi = 0$. Given any solution, we can apply a Lorentz transformation about the origin and get another. Now take one particular solution ψ_0, defined by



$\psi_0 = 1$ in the future light cone $t > r$ with $r = (x^2 + y^2 + z^2)^{1/2}$; $\psi_0 = -1$ in the past light cone, $t < -r$; $\psi_0 = t/r$ outside the light cone, $-r < t < r$. The diagram gives ψ_0 for all values of t and x.

This ψ_0 satisfies $\Box\psi = 0$. The result is obvious in the two regions $t > r$ and $t < -r$. Also it is easily checked in the region outside the light cone $-r > t < r$. Further, one notices that ψ_0 is continuous on the future light cone, $t = r$, and also on the past light cone, $t = -r$. One then finds that the equation $\Box\psi_0 = 0$ holds also on the light cones. To check this we use the result, for any spherically symmetrical ψ,

$$\Box\psi = \frac{\partial^2\psi}{\partial t^2} - \frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) \tag{6}$$

For ψ_0 we have, close to the future light cone,

$$r^2\frac{\partial\psi_0}{\partial r} = 0 \qquad \text{for } t > r \text{ and } -t \text{ for } t < r$$

This gives

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_0}{\partial r}\right) = 0 \qquad \text{for } t > r \text{ and } 0 \text{ for } t < r$$

Thus the second term of (6) vanishes for ψ_0, and the first obviously vanishes. A similar argument holds for the past light cone.

We may take ψ_0 and apply to it any Lorentz transformation about the origin. The result will be a ψ that is zero inside the future light cone, zero inside the past light cone, and nonzero outside the light cone. We can apply further Lorentz transformations to it and the result will always be a ψ that is zero within the future light cone, zero within the past light cone, and can be nonzero only outside the light cone. We can never get back to a ψ like ψ_0, with nonzero values inside the future and past light cones. We have the situation needed for a pathological representation, with ψ_0 providing both the ψ_A and ψ_B parts of the wave function, and those ψ's that have zero inside the past and future light cones providing the ψ_B part of the wave function. The ψ_B part can be generated by Lorentz transformations applied to ψ_0, but ψ_0, containing the ψ_A part, cannot be generated by a Lorentz transformation applied to a ψ_B part.

This is the simplest example of a pathological representation of the Lorentz group. It provides a natural value for the arbitrary coefficient λ that appears with a general pathological representation.

It may very well be that this pathological representation is essential for the physics of the future. Then one will be unable to make any important advance without it.

The question remains, whether the work that physicists are now engaged in, based on the ordinary representations of the Lorentz group, is of any value. While I believe that an important advance can be made only with the help of pathological representations, it may very well be that the present work will lead to secondary discoveries. The position is similar to that before 1925, when people were working with Bohr orbits. While an important advance was not possible, people were able to figure out correctly the notions of Bose statistics and Fermi statistics, quite important, although secondary discoveries. One should look out for comparable discoveries that may flow out from the present discussions.